Pergamon

Int. J. Solids Structures Vol. 34, No. 12, pp. 1517–1529, 1997

0020–7683/97 $17.00 + .00

PII: S0020–7683(96)00090–X

ON LENSKY'S HYPOTHESIS OF LOCAL DETERMINABILITY UNDER NONPROPORTIONAL CYCLIC LOADING

YANG XIAN-JIE and CAL LI-XUN
Institute of Applied Mechanics, Southwest Jiaotong University, Chengdu, Sichuan, 610031, P.R. China

(*Received* 27 *January* 1995; *in revised form* 12 *May* 1996)

Abstract—A series of experiments on steel 40 was conducted under nonproportional cyclic loadings. Using data from this experimental investigation, Lensky's hypothesis of local determinability is analysed in elastic and elasto-plastic conditions. It is shown that, under nonproportional cyclic loading paths, the above hypothesis does not hold in Ilyushin's total strain space, nor in the plastic strain space.

1. INTRODUCTION

With the development of engineering technology, many structural components are usually subjected to complex cyclic loadings. To determine the relation of stress vs strain under a complex loading history, Ilyushin (1963) proposed a general plasticity theory. In this theory, Ilyushin proposed the principle of fading memory, i.e. the current deformation behavior is not influenced by the remote history of deformation while it is affected by the nearest past history of deformation. Lensky (1962) proposed the hypothesis of local determinability: the delay angle θ_i ($i = 1, 2, \ldots 5$) extended by the current Frenet reference point $\{\mathbf{p}_i\}$ with respect to the next section of the deformation history and stress vectors entirely depends on the current value of the delay angle θ_j ($j = 1, 2, \ldots 5$), the intrinsic geometry of this section [the curvatures k_m ($m = 1, 2, 3, 4$)] and the arc length s of the proceeding trajectory. Lensky's hypothesis plays an important role in promoting the practical application of Ilyushin's theory. Shiratori *et al.* (1975) studied the behavior of the stress vector along cornered loading paths in the strain space by subjecting thin-walled tubular specimens of brass to a combined axial load with either torsion or internal pressure. They verified that Lensky's hypothesis holds under these loading conditions. Ohashi *et al.* (1981) also demonstrated experimentally that Lensky's hypothesis does hold for some bi-linear strain paths. Under nonproportional cyclic loadings, Ohashi *et al.* (1985) showed that the hypothesis does not hold under square strain paths at small equivalent strain amplitudes. Ning and Chen (1991) further revealed that, under nonproportional cyclic loadings, the hypothesis holds for some paths, and fails for some other paths. However, experimental data on the delay phenomenon are not yet extensive enough in order that they may be incorporated within the general theory of plasticity.

In the present paper, Lensky's hypothesis is discussed first for the case of pure elasticity. Then the results of experiments performed on thin-walled tubular specimens of steel 40 at room temperature under combined axial force and torque are described. The loading paths are along rhombic and triangular trajectories in the vector space of deviatoric strain in order to elucidate whether Lensky's hypothesis holds under out-of-phase strain cycles with an abruptly-changing principal strain rate direction. Finally, the modified Lensky's hypothesis in the plastic strain space is discussed under the aforementioned two kinds of nonproportional strain paths.

2. EXPERIMENTS

2.1. *Thin-walled tubular specimen*

Thin-walled tubular specimens were machined from a steel 40 bar of 40 mm diameter with a quenched and tempered treatment. The chemical composition of the material is: C: 0.42, Si: 0.24, Mn: 0.56, S: 0.006, P: 0.017, Fe: balance, in weight percent. The mechanical properties of the steel bar are: Young's modulus of 220 GPa, Poisson's ratio of 0.30, proof stress of 351.9 MPa, tensile strength of 598.7 MPa, elongation of 47.1%, reduction of area of 25.2%.

Experiments were conducted on a computer-controlled testing system comprising a DEC PDP11 microcomputer, a MTS809 servo- controlled electrohydraulic testing machine and a data acquisition system. Strain components in the specimen were evaluated using the measurements from an axial-torsional extensometer of 25 mm gage, which was positioned on the outer surface in the central part of the tubular specimen. The axial strain ε was defined as the gage length displacement divided by the original gage length. The shear strain γ was obtained by multiplying the ratio of the angle of twist to the gage length by the mean radius. Its relative error for the measurements of the axial strain and the torsional shear strain is 0.20%. The experiments were run at the strain rate $5\times10^{-4}\ \mathrm{s}^{-1}$ and at room temperature. The axial stress σ and the shear stress τ were assumed to be uniform over the thin wall.

2.2. *Stress and strain deviatoric vector space*

The definition of the axial-torsional subspace follows as a subspace of Ilyushin's five dimensional deviatoric vector space (Ilyushin, 1963). The stress vector is

$$\boldsymbol{\sigma} = \sigma_1\mathbf{n}_1 + \sigma_3\mathbf{n}_3 \tag{1}$$

where $\sigma_1 = \sigma, \sigma_3 = \sqrt{3}\tau$, and σ and τ are the axial and shear stresses, respectively.

$\mathbf{n}_1$ and $\mathbf{n}_3$ are the orthonormal base vectors in the stress space. Likewise, the strain vector is

$$\boldsymbol{\varepsilon} = \varepsilon_1\mathbf{n}_1 + \varepsilon_3\mathbf{n}_3 \tag{2}$$

where, $\varepsilon_1 = \varepsilon, \varepsilon_3 = \gamma/\sqrt{3}$, and ε and γ are the axial strain and engineering shear strains, respectively.

The plastic strain vector is heuristically defined as

$$\boldsymbol{\varepsilon}^{\mathrm{p}} = \varepsilon_1^{\mathrm{p}}\mathbf{n}_1 + \varepsilon_3^{\mathrm{p}}\mathbf{n}_3 \tag{3}$$

where $\varepsilon_1^{\mathrm{p}} = \varepsilon^{\mathrm{p}}, \varepsilon_3^{\mathrm{p}} = \gamma^{\mathrm{p}}/\sqrt{3}$, and ε^{p} and γ^{p} are the axial and shear plastic strains, respectively.

The Von Mises equivalent stress σ_e, then, can be written as

$$\sigma_e = |\boldsymbol{\sigma}| = \{(\sigma_1)^2 + (\sigma_3)^2\}^{1/2} \tag{4}$$

and the equivalent strain is given by

$$\varepsilon_e = |\boldsymbol{\varepsilon}| = \{(\varepsilon_1)^2 + (\varepsilon_3)^2\}^{1/2}. \tag{5}$$

The strain rate and plastic strain rate vectors may be defined as

$$\dot{\boldsymbol{\varepsilon}} = \dot{\varepsilon}_1\mathbf{n}_1 + \dot{\varepsilon}_3\mathbf{n}_3 \tag{6}$$

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = \dot{\varepsilon}_1^{\mathrm{p}}\mathbf{n}_1 + \dot{\varepsilon}_3^{\mathrm{p}}\mathbf{n}_3 \tag{7}$$

respectively, where ($\dot{\ }$) stands for the derivative with respect to time t. The arc length s of the strain path may be calculated from

$$s = \int_0^t |\dot{\varepsilon}| \, \mathrm{d}t. \tag{8}$$

The equivalent strain amplitude $\Delta\bar{\varepsilon}/2$ is taken as half of the maximum Euclidean norm between two arbitrary points A and B along the strain trajectory D in one cycle:

$$\Delta\bar{\varepsilon}/2 = 1/2 \, \mathrm{Sup}\,\{|\boldsymbol{\varepsilon}_{\mathrm{A}} - \boldsymbol{\varepsilon}_{\mathrm{B}}| : \quad \boldsymbol{\varepsilon}_{\mathrm{A}} \in \mathrm{D}, \boldsymbol{\varepsilon}_{\mathrm{B}} \in \mathrm{D}\}. \tag{9}$$

3. LENSKY'S HYPOTHESIS OF LOCAL DETERMINABILITY

Since Lensky's hypothesis will be discussed for the simple cases under rhombic and triangular straining paths in the two dimensional axial-torsional strain subspace, Lensky's hypothesis (1962) can be presented as

$$\frac{\partial \theta}{\partial s} = f(\theta, k, s), \quad \cos\theta = \boldsymbol{\sigma} \cdot \mathrm{d}\boldsymbol{\varepsilon}/(|\boldsymbol{\sigma}|\,|\mathrm{d}\boldsymbol{\varepsilon}|) \tag{10}$$

where θ is the delay angle extended by the strain rate and stress vectors, k is the curvature of the strain trajectory, $\mathrm{d}\boldsymbol{\varepsilon}$ denotes the total strain incremental vector.

For the convenience of discussion, the cyclic saturated data along cyclic strain trajectories were used. Since the arc length s along the cyclic saturated strain curve is long enough, eqn (10) can be simplified as

$$\frac{\partial \theta}{\partial s} = f(\theta, k). \tag{11}$$

When the hypothesis is discussed for the case of a straight-line trajectory, eqn (11) can be further simplified as:

$$\frac{\mathrm{d}\theta}{\mathrm{d}s} = f(\theta). \tag{11a}$$

4. LENSKY'S HYPOTHESIS UNDER ELASTIC DEFORMATIONS

Both Lensky (1962) and Ohashi (1981) verified the validity of the hypothesis under large straining. In the case of multi-linear trajectories discussed, the elastic strain is much less than the plastic one. On the other hand, under the nonproportional cyclic strain path the total strains often contain the large elastic components. When the elastic strain cannot be neglected, the results of Lensky (1962) and Ohashi *et al.* (1981) must be reconsidered. In Lensky's hypothesis, $\mathrm{d}\boldsymbol{\varepsilon}$ includes both elastic and plastic components. As a special case, a purely elastic behavior under the square and rhombic strain paths is investigated so that the effects of the elastic property on the validity of the hypothesis is verified.

It is known that the stress response of a purely elastic material is not path dependent, and that the elastic material has no memory. The relevant angle θ is uniquely determined for all strain paths specified as above. The θ–s relations for elastic material at the elastic Poisson's ratio of 0.3 under rhombic strain paths are shown in Fig. 1. The curves in Fig. 1(a) at the equivalent strain amplitudes of 0.3% and 0.5% show that, at the same angle θ,

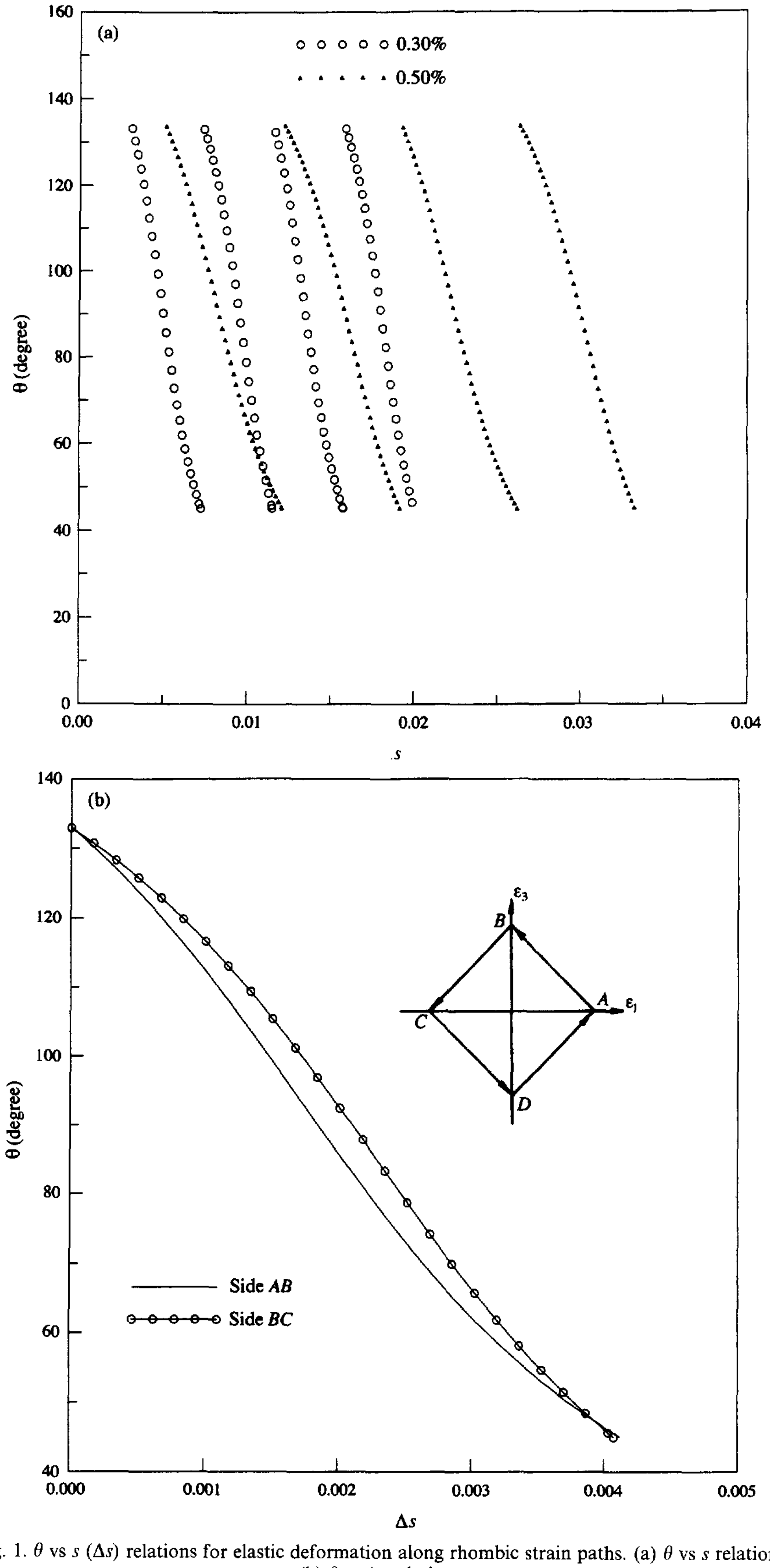

Fig. 1. θ vs s (Δs) relations for elastic deformation along rhombic strain paths. (a) θ vs s relation. (b) θ vs Δs relation.

the slope $|d\theta/ds|$ is smaller at 0.5% than that at 0.3%. These θ–s relations are significantly different even though the delay angles θ are the same at the corners of the strain trajectories at the two equivalent strain amplitudes. Meanwhile, even though the four sides on the rhombic strain path have the same arc length, all the angles at the four corners are right ones, and the θ–s relations are the same for the two opposite sides, such as AB and CD, the relations are different for the two neighboring sides like AB and BC. When the θ–s relation for side BC is transferred along the abscissa so that the increments Δs for the sides AB and BC are the arc lengths measured from sharp corner A and B, respectively, we find the two θ–Δs curves different, as shown in Fig. 1(b). Therefore, it is concluded that Lensky's hypothesis cannot be obeyed for the purely elastic behavior under the rhombic strain paths.

The θ–s relations for the square strain path are shown in Fig. 2. Here, the trajectory is multi-linear and its curvature k for each segment is zero. Fig. 2(a) corresponds to the θ–Δs relations for the neighboring sides AB and BC at the equivalent strain amplitude of 0.30%. The θ–s relation for side BC is transferred along the abscissa to the θ–Δs relation where Δs is the arc length measured from sharp corner B. Obviously, the values of $|d\theta/ds|$ for the two sides are different at the same θ. Figure 2(b) presents the θ–s relations at the equivalent strain amplitudes of 0.3 and 0.5%, respectively. Even though the θ–s relations for the opposite sides at a given equivalent strain amplitude are the same, the ones at different equivalent strain amplitudes are different. At the same θ, the $|d\theta/ds|$ is larger at the small equivalent strain amplitude 0.3% than that at the large amplitude 0.5%. Consequently, Lensky's hypothesis cannot be obeyed for purely elastic behavior along the square strain path.

The difference between the θ–s relations for two neighboring sides is mainly caused by the fact that the elastic Poisson's ratio is not equal to 0.5. The difference among the θ–Δs relations under different equivalent strain amplitudes is due chiefly to the purely elastic property. For this reason, Lensky's hypothesis cannot be obeyed under purely elastic nonproportional cyclic strain paths.

5. LENSKY'S HYPOTHESIS FOR ELASTO-PLASTIC BEHAVIOR

Elasto-plastic responses of materials under nonproportional cyclic straining are more complicated than elastic ones. In general, plastic strains occur when total strains are large enough. When the equivalent strain amplitude is small, the elastic strain is the most part of the total strain and thus the θ–s relation is controlled to a great degree by the elastic rule since the direction of the stress vector is approximately determined by the elastic theory. When the equivalent strain amplitude is large, the elastic strain range is negligible in comparison with the plastic strain one and thus the θ–s relation is controlled to a smaller degree by the elastic rule with the Poisson's ratio ν approaching 0.5. Based on the above theoretical analysis, it seems that when the equivalent strain amplitude is small, Lensky's hypothesis cannot be valid.

The validity of Lensky's hypothesis is experimentally examined for nonproportional cyclic loading within the elasto-plastic range. Ohashi *et al.* (1985) showed that Lensky's hypothesis for 316 stainless steel does not hold under the square strain path at a small equivalent strain amplitude, but the θ–Δs relations approach a unique curve with the increase of the equivalent strain amplitude. In other words, when the equivalent strain amplitude is large enough, the above hypothesis may be obeyed for 316 stainless steel under square paths.

The validity of Lensky's hypothesis depends on the uniqueness of the θ–Δs curve for the multi-linear paths with different shapes and various strain amplitudes. In the following the validity of Lensky's hypothesis for steel 40 under rhombic and triangular strain paths is investigated.

5.1. *Rhombic strain trajectories*

Figure 3 shows rhombic strain trajectories in the deviatoric strain plane ($\varepsilon_1, \varepsilon_3$) obtained from the out-of-phase experiments with $\Delta\varepsilon_1/2 = 0.47\%$ and six different $\Delta\varepsilon_3/2$. Here, θ_a and θ_b denote the abruptly-changing angles at the sharp corners along the trajectories. As $\Delta\varepsilon_3/2$

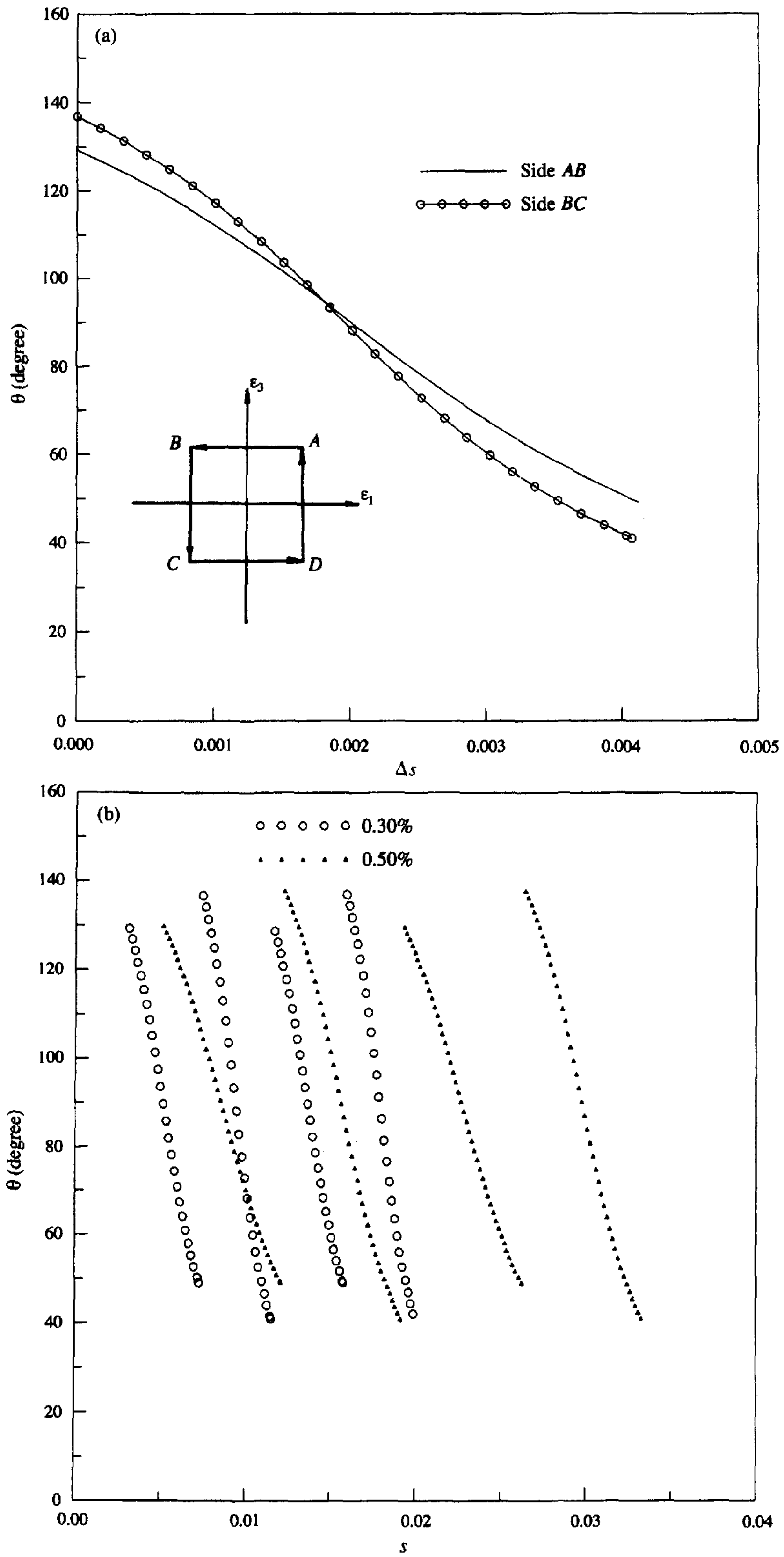

Fig. 2. θ vs s (Δs) relations for elastic deformation along square strain paths. (a) θ vs Δs relation. (b) θ vs s relation.

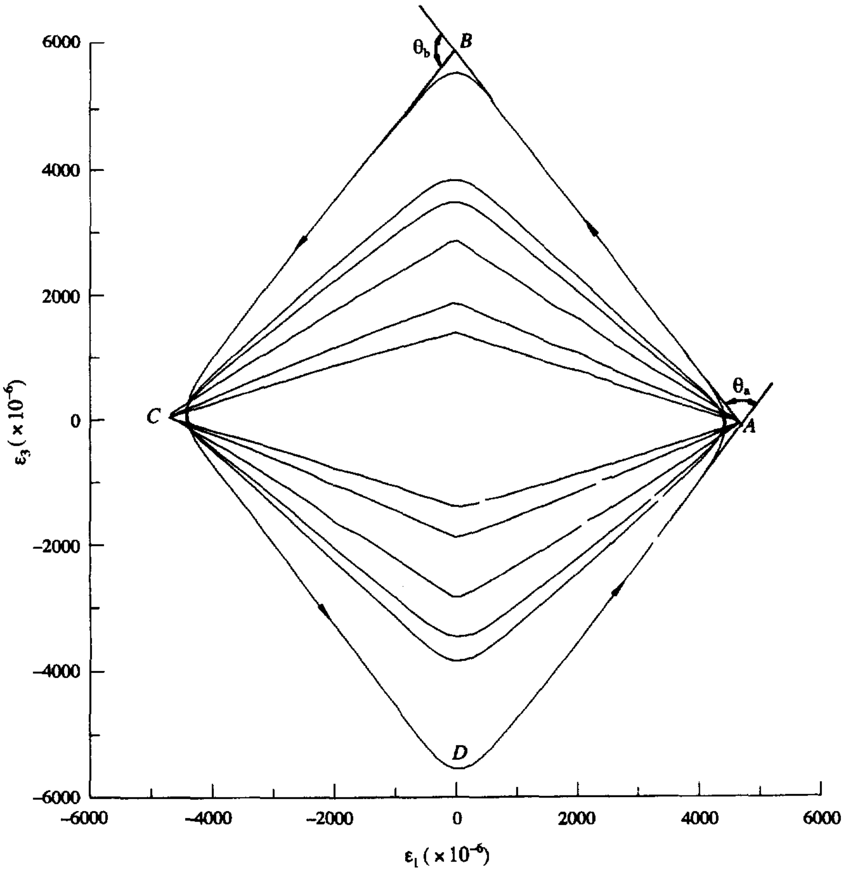

Fig. 3. The experimental rhombic paths on steel 40.

increases, θ_a becomes smaller and θ_b larger. The arc lengths for the four sides are the same for each $\Delta\varepsilon_3/2$, and the equivalent plastic amplitudes are much larger than elastic ones under these strain trajectories. The θ–$\Delta s'$ curves for steel 40 are shown in Figs 4–6. Figure 4 shows that, for side AB, the value of $|d\theta/ds|$ at a given θ of more than 60° increases with a decrease of the $\Delta\varepsilon_3/2$. When $\Delta s'$ is large and θ is small, the θ–$\Delta s'$ curves for the six rhombic strain paths tend to coincide with each other. Figure 5 indicates that when $\Delta s'$ is small, the θ–$\Delta s'$ curves for side BC agree with each other, on the other hand, when $\Delta s'$ is larger than 0.2%, $|d\theta/ds|$ at the same θ decreases with the increase of $\Delta\varepsilon_3/2$. Figure 6 presents the θ–$\Delta s'$ curves for the sides AB and BC with six different $\Delta\varepsilon_3/2$. Evidently, the θ–$\Delta s'$ curves for the side AB are quite different from those for the side BC. Furthermore, these curves do not agree with each other by translating along the $\Delta s'$ axis. When the locus of strain vector turns a corner, the value of θ has an abrupt increase. Theoretically the increment of θ equals θ_a (or θ_b) at the corner since the stress vector does not follow the change in the direction of the strain trajectory immediately. Because the corners along the practical strain path are not sharp and have finite curvatures, the sudden increment of θ after the corner is smaller than the angle θ_a (or θ_b). Ohashi *et al.* (1981) confirmed that the current curvature of trajectory has little influence on the subsequent θ–Δs relation for subsequent straight-line trajectory. From Figs 4–6, it is concluded that the plastic behavior along rhombic strain trajectories does not obey Lensky's hypothesis.

5.2. *Triangular strain trajectories*

Under the loadings along the equilateral triangular strain paths with three equivalent strain amplitudes of 0.2, 0.3 and 0.38%, the θ–Δs relations for steel 40 are shown in Figs 7 and 8. Figure 7 demonstrates that the θ–Δs curves for the three sides of the triangular trajectory at the equivalent strain amplitude of 0.30% are approximately identical when the three θ–s curves are transferred to the θ–Δs ones where Δs is measured from the

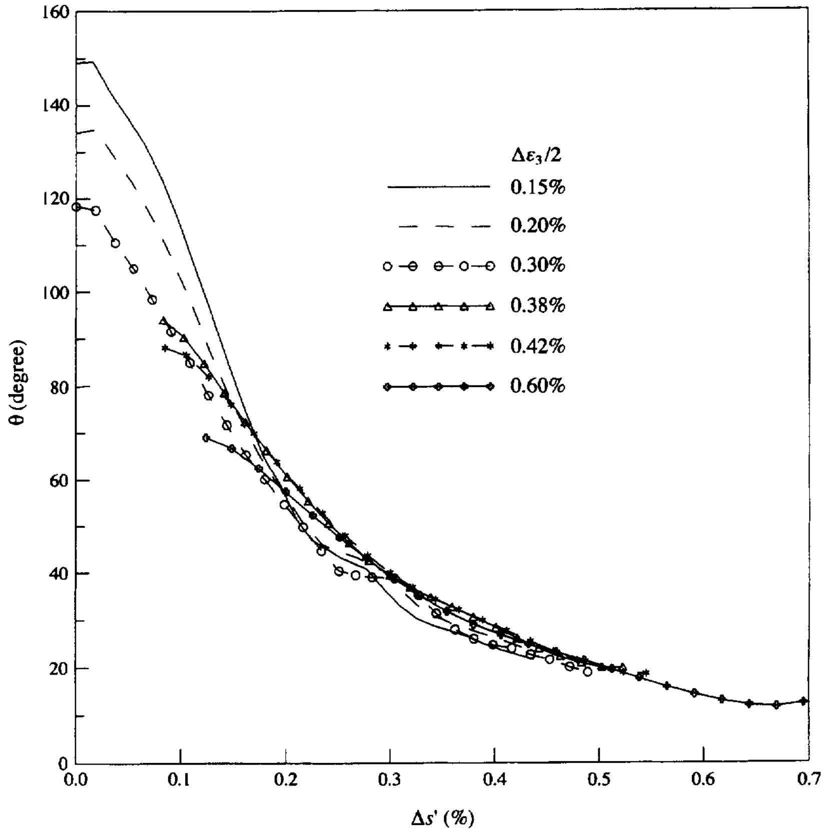

Fig. 4. Relation θ vs $\Delta s'$ for side AB ($\Delta s' = \Delta s - x$. Δs: arc length after the corner, x: arc length transferred rightward).

respective corner. The same conclusion can be obtained for the equilateral triangular paths with the equivalent strain amplitudes of 0.20 and 0.38%. Hence, the assumption of isotropy is considered valid. Figure 8 shows that the θ–$\Delta s'$ relations for side AB, under the above three equivalent strain amplitudes, depend significantly on the equivalent strain amplitude. As the equivalent strain amplitude increases, the value of $|d\theta/ds|$ at the same θ decreases for the $\Delta s'$ less than 0.1%. This indicates that the values of $d\theta/ds$ depend not only on the delay angle θ, but also on the equivalent strain amplitude. Therefore, Lensky's hypothesis cannot be obeyed. However, the differences among the θ–$\Delta s'$ curves tend to be small with the increase of equivalent strain amplitude. This result is in agreement with that of Ohashi *et al.* (1985).

5.3. *Lensky's hypothesis in the plastic strain space*

In axial-torsional plastic strain subspace, θ^p and s^p are defined as

$$\cos\theta^p = \frac{\boldsymbol{\sigma}\cdot d\boldsymbol{\varepsilon}^p}{|\boldsymbol{\sigma}|\,|d\boldsymbol{\varepsilon}^p|} \tag{12}$$

$$s^p = \int_0^t |\dot{\boldsymbol{\varepsilon}}^p|\,dt. \tag{13}$$

In the paper, the validity of Lensky's hypothesis in the plastic strain space is examined by means of the evolution of the delay angle θ^p between the tangent to the plastic strain trajectory and the stress vector since the property of plastic flow can be revealed quite well in the plastic space. It is difficult to examine directly the validity of the hypothesis in this case. Axial-torsional tests are performed in general by the total strain control since a

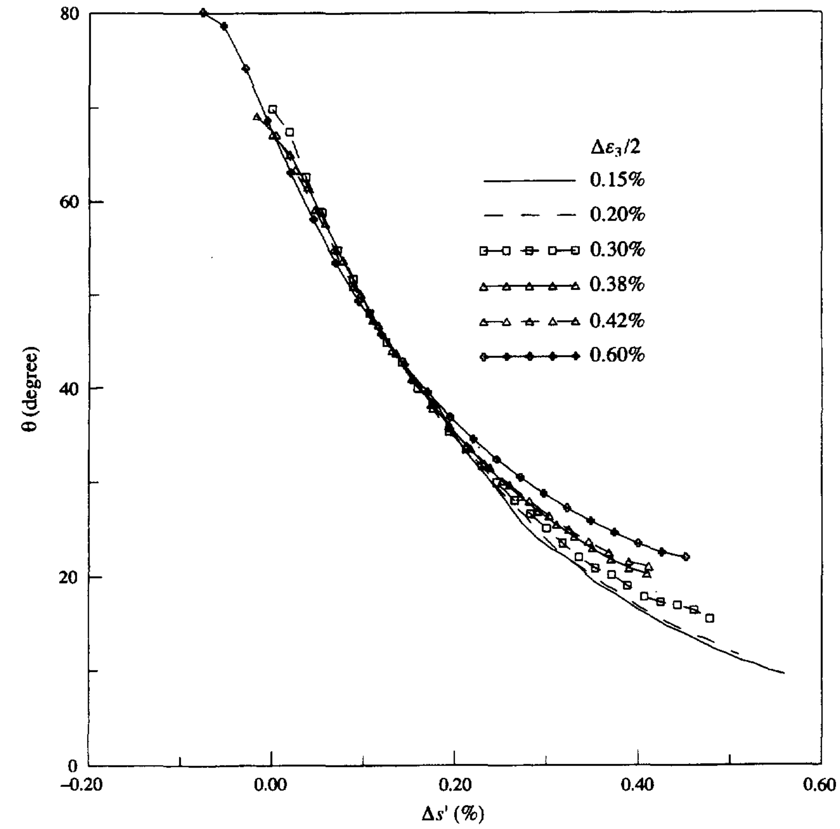

Fig. 5. Relation θ vs $\Delta s'$ for side BC ($\Delta s' = \Delta s - x$. Δs: arc length after the corner, x: arc length transferred rightward).

plastic strain controlled test is not easy to carry out. The experimental results under the nonproportional cyclic multi-linear trajectory controlled by total strain show that the response is a curvilinear plastic strain trajectory in the plastic strain space and its curvature changes continuously along each segment. Obviously, $d\theta^p/ds^p$ is influenced by θ^p and the curvature θ^p. Since to demonstrate quantitatively the modified Lensky's hypothesis in the plastic strain space is a difficult task, we shall analyse the hypothesis qualitatively.

Assuming the plane plastic strain trajectory as shown in Fig. 9, the angle θ^p is defined as

$$\theta^p = \theta_L - \theta_1 \tag{14}$$

where θ_1 is the angle between the stress vector and the base vector $\mathbf{n}_1$, and θ_L the angle between the tangent to the plastic strain trajectory and the base vector $\mathbf{n}_1$. Thus,

$$\frac{d\theta^p}{ds^p} = k^p - \frac{d\theta_1}{ds^p} \tag{15}$$

where

$$k^p = \frac{d\theta_L}{ds^p} \tag{16}$$

is the curvature of the plastic strain trajectory.

When the cycles are saturated, the modified Lensky's hypothesis may be expressed as

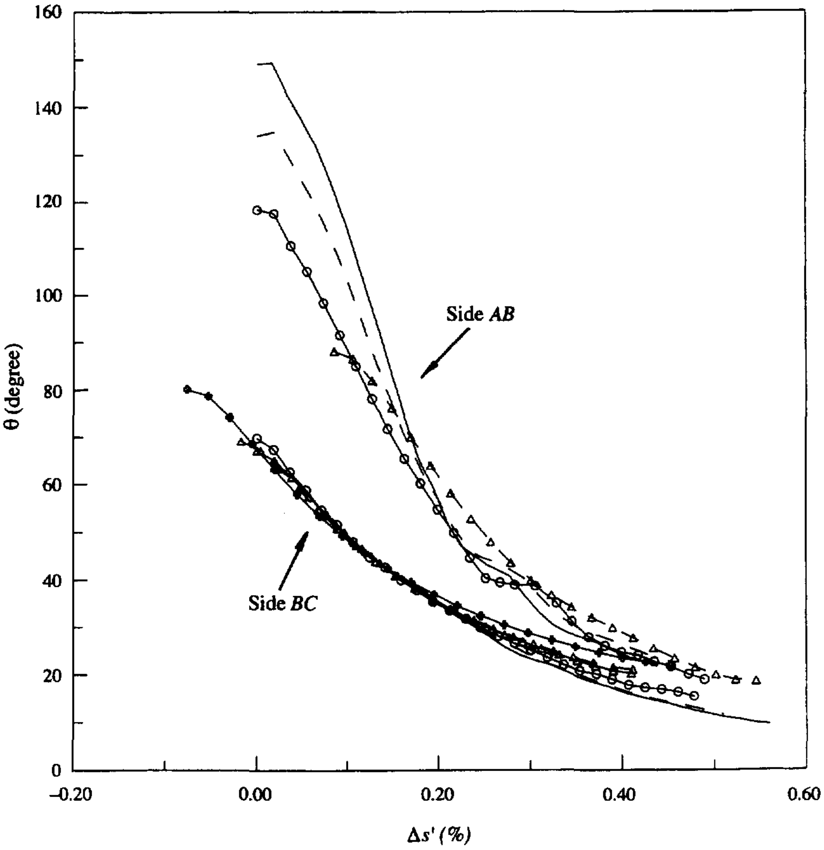

Fig. 6. The θ–$\Delta s'$ relations for side AB and BC on rhombic paths.

$$\frac{d\theta^p}{ds^p} = f(\theta^p, k^p). \tag{17}$$

Equation (17) states clearly that, if Lensky's hypothesis is valid for a specific material, $d\theta^p/ds^p$ must be a single-valued function of θ^p and k^p under an arbitrary plastic strain trajectory at cyclic saturation.

The abruptly-changing direction of plastic flow at a sharp corner will cause stress relaxation, and the plastic strain response cannot be calculated accurately in the vicinity of the sharp corner along the trajectory because of test errors. Therefore, the values of the curvature k^p and $d\theta_1/ds^p$ in the vicinity of a sharp corner from numerical calculations are not accurate. Except for the short range near the corner, the values of k^p, $d\theta^p/ds^p$ and $d\theta_1/ds^p$ by numerical calculation satisfy eqn (15) quite accurately and thus the calculated values of k^p, θ^p, $d\theta^p/ds^p$ at some distance from a sharp corner may be considered to be accurate enough. Figures 10 and 11 provide, through the curves θ^p–$d\theta^p/ds^p$ and θ^p–k^p, the values of $d\theta^p/ds^p$ and k^p corresponding to θ^p along the strain trajectory.

Figure 10 shows that, for the two rhombic trajectories at the $\Delta\varepsilon_3/2$ of 0.15 and 0.60%, the θ^p–$d\theta^p/ds^p$ curves for the θ^p smaller than 20° are almost identical and the θ^p–k^p ones at the two $\Delta\varepsilon_3/2$ are almost the same. However, when the abruptly-changing θ is an obtuse angle, the θ^p is generally large along the subsequent loading path. At the same θ^p and along the segment where the θ^p–k^p curves for the two rhombic trajectories are approximately identical, the derivative $|d\theta^p/ds^p|$ at the $\Delta\varepsilon_3/2$ of 0.15% is significantly larger than that at 0.60%. So, the values of $|d\theta^p/ds^p|$ are different at the given θ^p and k^p for the two rhombic trajectories when θ^p is larger than 20°. Therefore, the modified hypothesis cannot be obeyed along the rhombic strain trajectories.

Figure 11 shows the θ^p–$d\theta/ds^p$ and θ^p–k^p curves for two triangular strain trajectories at the equivalent strain amplitudes of 0.30 and 0.38%. Even though the θ^p–k^p curves in the

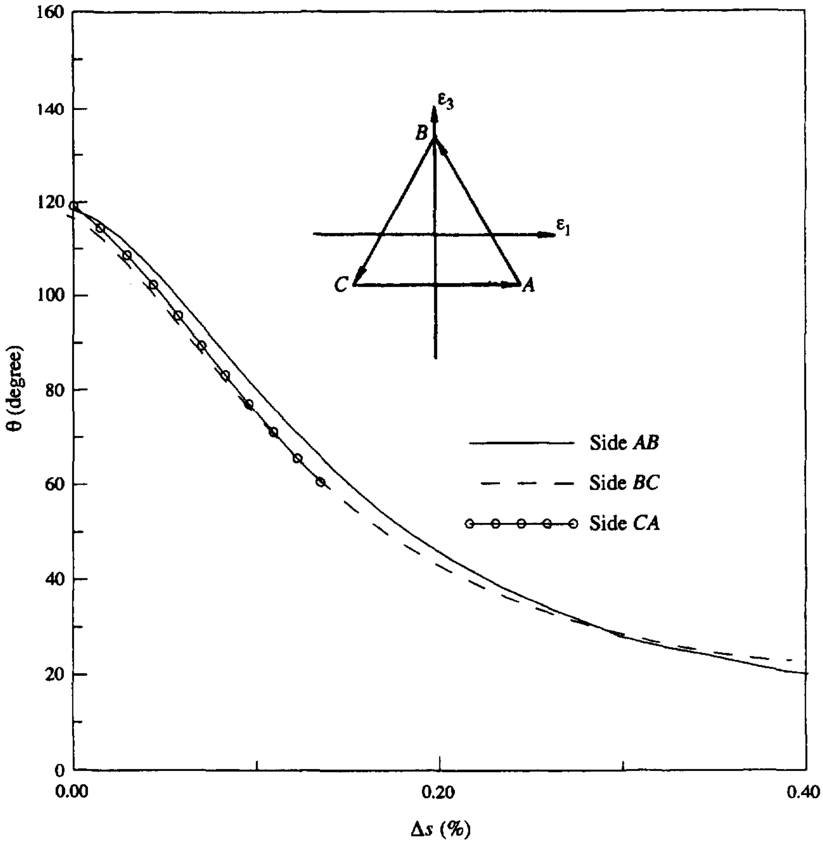

Fig. 7. Relation θ vs Δs after the corners on the equilateral triangular trajectory under equivalent strain amplitude of 0.30%.

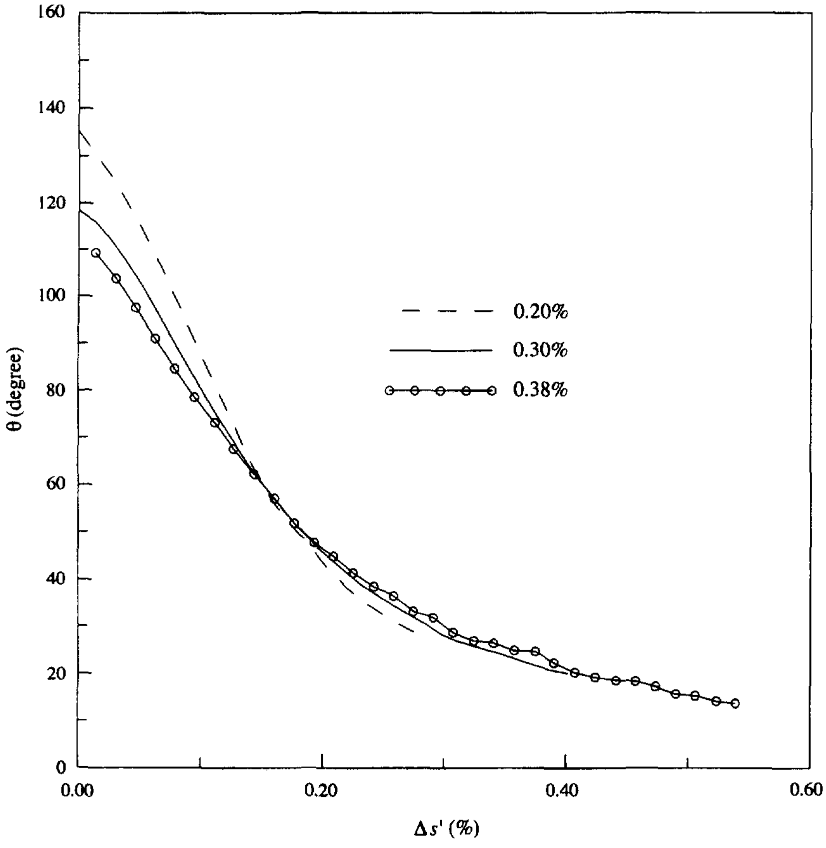

Fig. 8. Relations θ vs $\Delta s'$ for side AB on the equilateral triangular trajectories at three equivalent strain amplitudes of 0.20, 0.30, 0.38%, respectively ($\Delta s' = \Delta s - x$. Δs: arc length after the corner, x: arc length transferred rightward).

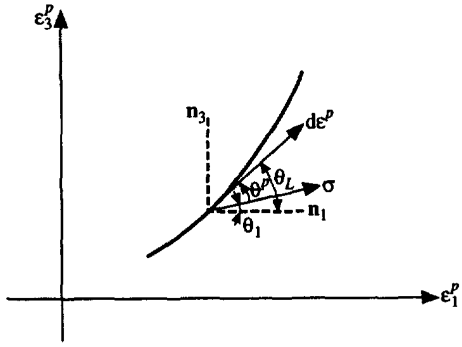

Fig. 9. The schematic diagram of stress and plastic strain vectors.

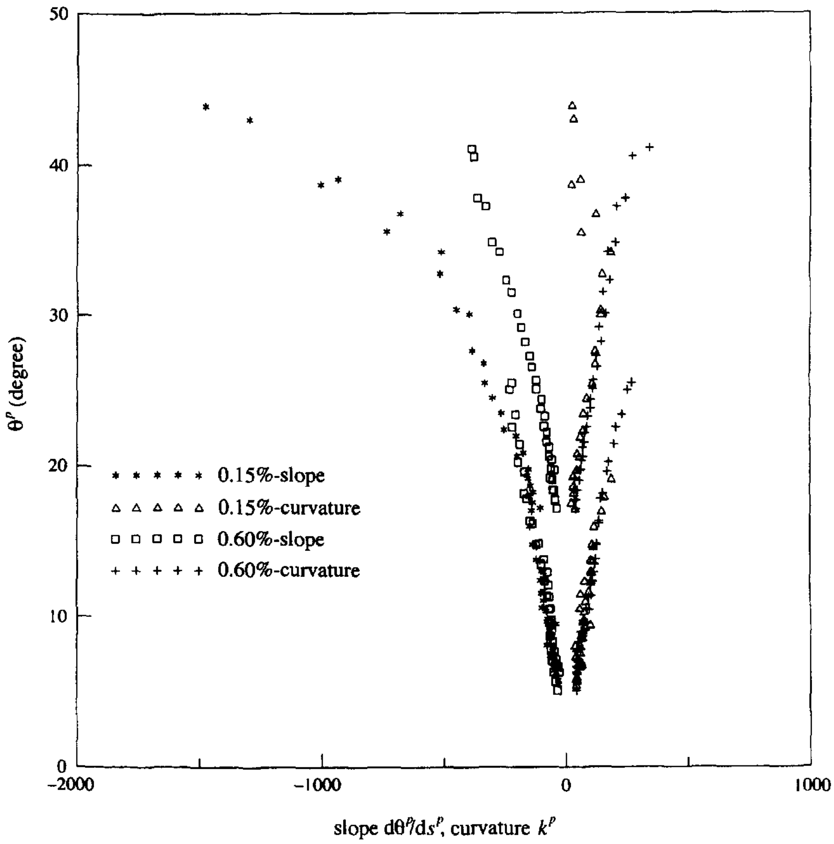

Fig. 10. The θ^p–k^p and θ^p–$d\theta^p/ds^p$ relations for rhombic trajectories.

cases along the bottom side CA shown in Fig. 7 are approximately the same, the corresponding θ^p–$d\theta^p/ds^p$ curves are different, i.e. at the same values of θ^p and k^p the value of $|d\theta^p/ds^p|$ at the equivalent strain amplitude of 0.38% is smaller than that at 0.30%, especially at large θ^p.

The above results again demonstrate that the modified Lensky's hypothesis in the plastic strain space is not true along the triangular strain trajectories.

6. CONCLUSION

The validity of Lensky's hypothesis of local determinability for steel 40 under the purely elastic and the elasto-plastic nonproportional cyclic loadings is discussed in both the total strain space and the plastic strain space. The following results are obtained:

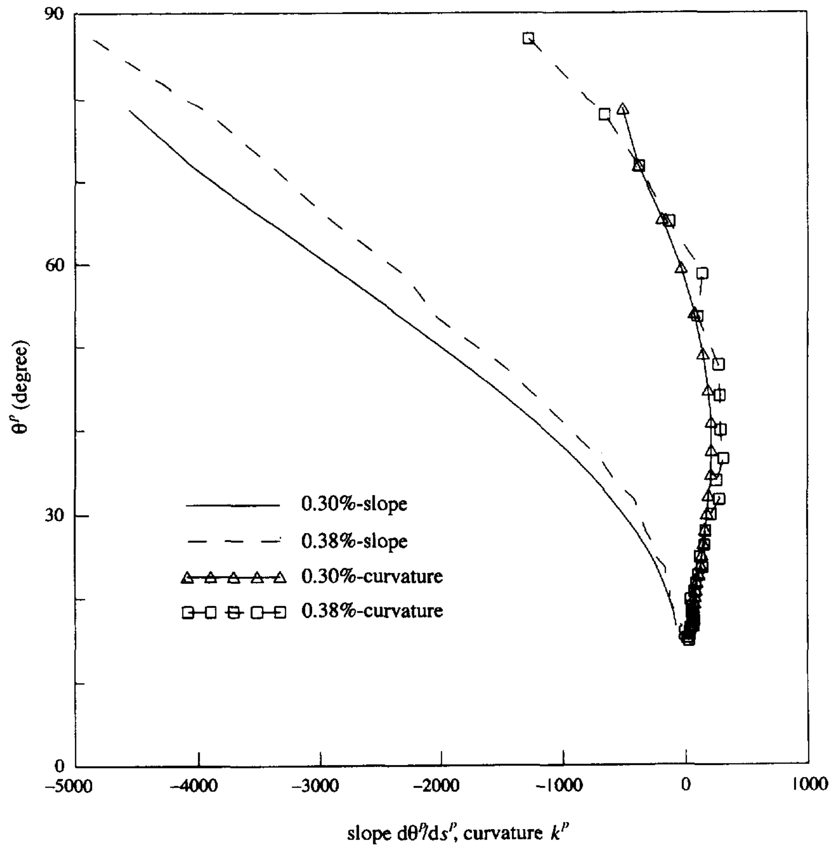

Fig. 11. The θ^p–k^p and θ^p–$d\theta^p/ds^p$ relations for side CA of equilateral triangular strain trajectories.

(1) The hypothesis cannot be obeyed for the cyclic deformation behavior that is controlled mainly by elastic properties.

(2) The hypothesis cannot be obeyed under nonproportional elasto-plastic rhombic and triangular strain paths for steel 40 when equivalent strain amplitude is small.

(3) In the cases with obtuse veering angles, Lensky's hypothesis for steel 40 cannot be obeyed under rhombic and triangular strain trajectories at different equivalent strain amplitudes, even in its modified form in the plastic strain space.

The above results imply that both eqns (1) and (17) cannot hold. The information provided by eqns (10) and (17) is not enough to determine the stress response of material under a nonproportional cyclic loading. Consequently, Lensky's hypothesis does not have a general validity under the nonproportional cyclic loading.

Acknowledgement—Financial support on the contract of 19402015 from NSFC is fully acknowledged.

REFERENCES

Ilyushin, A. A. (1963) Plasticity. Foundations of general mathematical theory. *Akad. Nauk.* SSSR, Moscow (in Russian).

Lensky, V. S. (1962) Local hypothesis in the theory of plasticity. Akad. Nauk. SSSR, *Otd. Tekh. Nauk. Mekh. Mashinostroenie*, No. 5, 154–158 (in Russian).

Ning, J. and Chen, X. (1991) On the properties of plastic flow of material under nonproportional cyclic loading. *International Journal of Solids and Structures* **28**, 403–412.

Ohashi, Y., Kurita, Y., Suzuki, T. and Tokuda, M. (1981) Experimental examination of the hypothesis of local determinability in the plastic deformation of metals. *Journal of the Mechanics and Physics of Solids* **29**, 51–67.

Ohashi, Y., Tanaka, E. and Ooka, M. (1985) Plastic deformation behavior of type 316 stainless steel subject to out-of-phase strain cycles. *ASME Journal of Engineering Material Technology* **107**, 286–292.

Shiratori, E., Ikegami, K. and Kaneko, K. (1975) Stress and plastic strain increment after corners on strain paths. *Journal of the Mechanics and Physics of Solids* **23**, 325–334.